Newtonian solutions, characterized by

$$\eta_{\text{Rel.}} \leq 1.2 \qquad (3)$$

For the ET-597 solutions of Astarita, Greco, and Nicodemo's study, Equation (3) limits the maximum allowable concentration to 30 ppm.[*] For Separan AP-30 (Dow), the allowable concentration is 125 ppm. (*4*); for Polyox WSR-301 (Union Carbide) 100 ppm, and for Polyox N-750 600 ppm. (*5, 6*). Under the restriction of Equation (3), β may be written as f/f_s, where f_s is the friction factor based on the solvent Reynolds number, with a maximum error of less than 5%.[†] Equation (1) is then

Equation (1) is then

$$\frac{f}{f_s} = \beta\,(N_{De}) = \beta\left(\frac{\theta {\eta_s}^{0.75}}{\eta^{0.75}}\,\frac{U}{D}\,{N_{Re_s}}^{0.75}\right) \qquad (4)$$

where N_{Re_s} is the solvent Reynolds number. Using an argument identical with that leading to Equation (2), we find

$$\frac{f}{f_s} = \beta'\left(\frac{\omega_s}{\omega_{s,\beta}}\right) \qquad (5)$$

where $\omega_s = U/D\;{N_{Re_s}}^{0.75}$, and $\omega_{s,\beta}$ is the value of ω_s at an arbitrary value of f/f_s. The point here is that, according to the theory, an infinite number of reduced universal curves exist, corresponding to different values of $\omega_{s,\beta}$ [or $\omega_{,\beta}$ in Equation (2)]. It would seem advantageous, in general, to utilize a reduced curve with a more easily attainable value of β, say 0.80. In this case, we have

$$\frac{f}{f_s} = \beta''\left(\frac{\omega_s}{\omega_{s,0.8}}\right) \qquad (6)$$

Equation (6) offers the following advantages in use as compared to Equation (2):

(1) A value of β'' less than unity corresponds to an actual reduction in drag with respect to the solvent.

(2) An accurate value of the solution viscosity is not required; we need only know Equation (3) is satisfied.

(3) $\omega_{s,0.8}$ would be, in general, easier to obtain than $\omega_{,0.5}$.

These comments, of course, presuppose the basic aspects of the theory to be correct. Further experiments are needed on this point. The essential diameter dependence of drag reduction predicted by Equations (2) and (6) has been well verified (*7*).

NOTATION

D = tube diameter
f = measured friction factor
f_0 = friction factor predicted from Newtonian correlations, using solution viscosity to obtain N_{Re}
f_s = friction factor predicted from Newtonian correlations, using solvent viscosity to obtain N_{Re}
N_{De} = Deborah number, defined following Equation (1)
$N_{De,0.5}$ = value of Deborah number corresponding to $f/f_0 = 0.5$
N_{Re} = Reynolds number, based on solution viscosity
N_{Re_s} = Reynolds number, based on solvent viscosity
U = average velocity
β = function defined in Equation (1)
β' = function defined in Equation (2)
β'' = function defined in Equation (5)
η = solution viscosity
η_s = solvent viscosity
$\eta_{\text{Rel.}}$ = η/η_s
θ = characteristic time of solution
ω = frequency, defined following Equation (2)
$\omega_{,0.5}$ = value of ω at $\beta' = 0.5$
ω_s = frequency, defined following Equation (5)
$\omega_{s,\beta}$ = value of ω_s at an arbitrary value of β

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[*] This figure is based on Oliver's value of $\eta_{\text{Rel.}} = 1.70$ for a freshly prepared 0.01% solution (3). For a slightly degraded solution, the allowable concentration would increase significantly.

[†] With $\eta_{\text{Rel.}} \leq 1.4$, this increases to about 9%.

Similarity Transformations for Continuous Cylindrical Surfaces in Axial Motion

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The article by Vasudevan and Middleman (2) makes a valuable contribution by presenting the velocity, temperature, and concentration profiles for a fluid surrounding a continuous circular cylinder in axial motion. It appears, though, that the assumption

$$\pi_i = \pi_i(\eta) \qquad (1)$$

which was used in deriving the equation

$$\frac{d^2\pi_i}{d\eta^2} + \frac{\Lambda_i}{\eta}\,f\,\frac{d\pi_i}{d\eta} = 0 \qquad (2)$$

is not justified.

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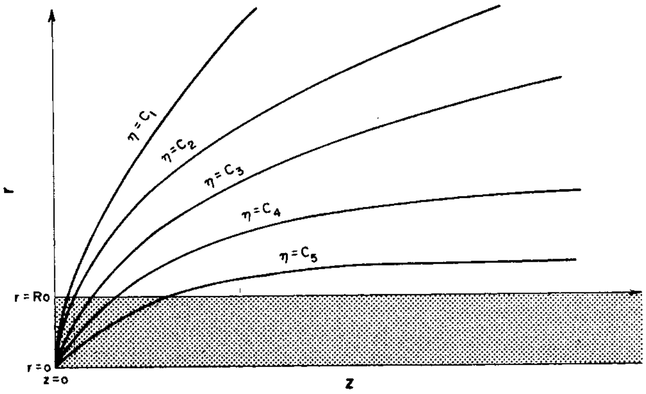

Fig. 1. Lines of constant η on an *r*, *z* coordinate system.

Figure 1 shows lines of $\eta = \frac{v}{4\nu}\frac{r^2}{z}$ = constant superimposed over the original *z*, *r* coordinate system. The constants C_i that are listed follow the relation

$$C_1 > C_2 > C_3 > C_4 > C_5 \qquad (3)$$

The intersection of lines of constant η with the line $r = R_o$ corresponds to η_o, the cylinder boundary.

If, for example, we consider π_v, the appropriate boundary condition is

$$\pi_v = 1 \text{ at } \eta = \eta_o \qquad (4)$$

However, if π_v is a function of η alone, then π_v must equal 1 at $\eta = C_1, C_2, \ldots, C_5$ since these lines cross the line $r = R_o$. The quadrant $r \geqq 0$, $z \geqq 0$ is, of course, densely filled with lines of constant η and all of these lines intersect $r = R_o$. Hence, the assumption that π_v is a function of η alone implies $\pi_v = 1$ everywhere. The above statement introduces a logical contradiction and one is forced to conclude that π_v is not a function of η alone. Thus, the transformation leading to Equation (2) is incorrect and Equation (2) itself is not in proper form.

It should further be noted that the limiting value of C_f for large η should approach the value calculated for a moving continuous flat plate since the effects of curvature are not important in this region. According to Sakiadis (*1*)

$$C_f \text{ is proportional to } z^{-1/2} \qquad (5)$$

whereas Vasudevan and Middleman's calculations seem to approach a functional form of

$$C_f \text{ is proportional to } z^{-1} \qquad (6)$$

NOTATION

C_f = local friction coefficient
$C_1, \ldots, C_5$ = constants
D_o = cylinder diameter, L
f = function in Equation (2)
r = radial coordinate, L
R_o = cylinder radius, L
V = velocity of cylinder, L/t
v_z = axial component of velocity, L/t
z = axial coordinate, L

Greek Letters

$\eta = \frac{Vr^2}{4\nu z}$ = similarity variable
ν = kinematic viscosity, L^2/t
Λ = constant in Equation (2)
π = function in Equation (2)

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Comment on the Communication of Fox and Hagin

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Our claim of a similarity transform for Equation (11) is in error. Inspection of Equation (14), in which f is defined as

$$f = -\gamma + \eta_0 + \frac{1}{\Lambda_i} + \int_{\eta_0}^{\eta} \Pi_v d\eta$$

indicates that for small η_0, say $\eta_0 < 0.01$, f may be only weakly dependent on η_0, especially near the jet surface. The good agreement between our theory and the work of Sakiadis in this range seems to bear this out.

The magnitude of the error in our results for $\eta_0 > 0.01$ is difficult to assess. As Fox and Hagin (see above) point out, and as is evident in our figures, our theory deviates significantly from Sakiadis' results for large η_0. As we point out in Paragraph 1, page 618, of citation 2 above, Sakiadis states that his method does not work well for large η_0. At the present time there is no accurate theory for this region.

It is fortunate that application of these results to fiber spinning will normally be such that $\eta_0 < 0.01$. Our application in jet stability studies has sometimes gone above $\eta_0 = 0.1$, but below $\eta_0 = 1$.

We wish to express our appreciation to Professor Cole of Clarkson College for his helpful comments on this matter.